MACADAM

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Asymptotic Numerical Method for Multiscale Buckling Analysis of Heterogeneous Materials

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  - Elastic microbuckling of a heterogeneous material
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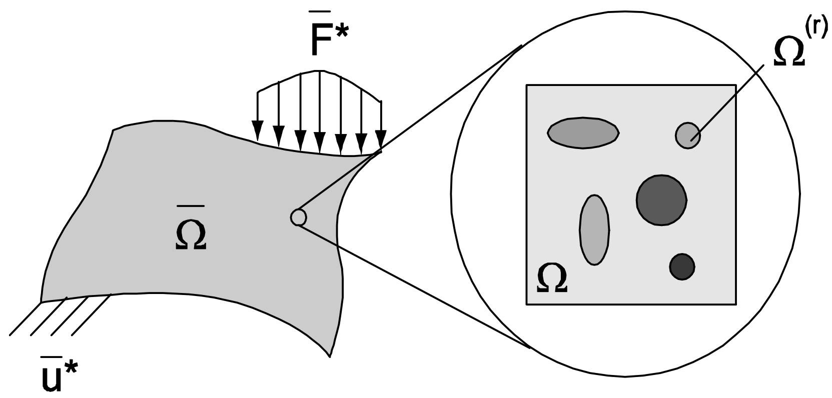
Introduction
Numerical homogenization of the nonlinear problems

- **The fully meshed model:** *the whole structure is meshed, including all heterogeneities;* ***Ladevèze et al. (2001)***.
- **The multiscale finite element method (FE^2):** *This technique simply correlates the micro structural response with the overall macroscopic behaviour, irrespective of the deformation level or the material behaviour;* ***Smit 1998, Terada 2001, Ghosh 2001, Feyel 2000-2003, Yvonnet 2007.***



Introduction

Using homogenization to deal with instabilities

<u>The instability phenomena</u> **in the context of multiscale homogenization analysis of heterogeneous materials are of *<u>structural (macro) and/or material (micro)</u>* types.**

Abeyaratne and Triantafyllidis (1984) **studied the macroscopic instability of a finitely strained periodic porous elastomer with a strictly rank-one convex matrix by homogenizing the tangent moduli.**

Miehe et al. (2002): **Investigation of the properties and the interaction of two coupled minimization principles: *<u>the principle of minimum potential energy of the macrostructure and the principle of minimum average energy of the microstructure.</u>***

Introduction
Our homogenization approach

We combine *the multilevel finite element method* (FE^2) and *the asymptotic numerical method* (ANM).

The principle of the ANM is to build up the solution branch in the form of power series that are truncated at rather large orders.

Introducing such expansion in the nonlinear problems at both macro and micro scales, it results a series of linear problems.

The superposition principle is applied on each linear problem.

The advantages with the ANM:

- The localization tensor is constructed for each asymptotic step;
- High order approximation;

Introduction

Our homogenization approach

- Only one decomposition of the tangent matrix per step;
- Large asymptotic steps;
- No iteration;
- Estimate a posteriori the length of each step of calculation in contrary to the method of Newton-Raphson.

→

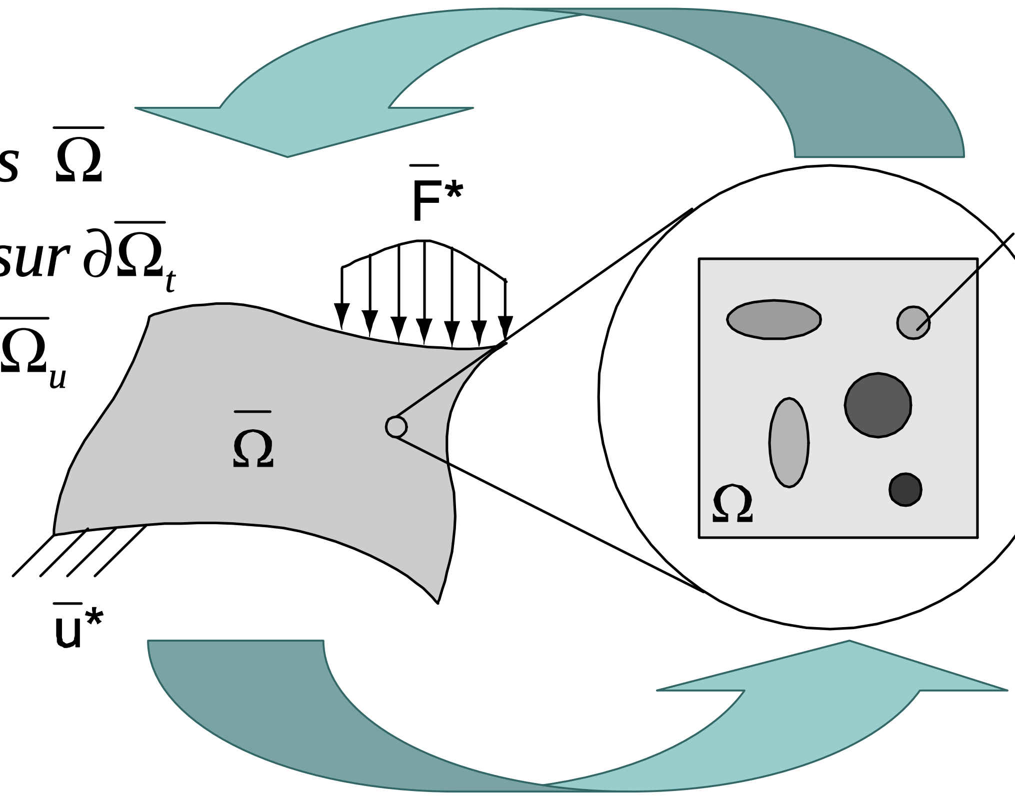
- Reduction of calculation time;
- Following the complicated responses.

Micro-macro problem formulation

$$\overline{\mathbf{P}} = \frac{1}{|\Omega|}\int_\Omega \mathbf{P}\,d\Omega \qquad \overline{\mathbf{F}} = \frac{1}{|\Omega|}\int_\Omega \mathbf{F}\,d\Omega$$

$$\nabla.\overline{\mathbf{P}} = 0 \quad dans \quad \overline{\Omega}$$

$$\begin{cases} \overline{\mathbf{P}}.\overline{N} = \Lambda \overline{F}^* & sur\, \partial\overline{\Omega}_t \\ \overline{u} = \overline{u}^* & sur\;\partial\overline{\Omega}_u \end{cases}$$



$$\nabla.\mathbf{P} = 0 \quad dans \quad \Omega$$

$$\mathbf{S} = \mathbf{C}^{(r)} : \boldsymbol{\gamma}$$

Boundary conditions in the microscopic scale

Micro-macro problem formulation

Boundary conditions in the microscopic scale

$$x = \overline{\mathbf{F}} X + w$$

$$\mathbf{F} = \overline{\mathbf{F}} + \nabla w$$

$$\overline{\mathbf{F}} = \frac{1}{|\Omega|}\int_{\Omega} \overline{\mathbf{F}} X \, d\Omega + \frac{1}{|\Omega|}\int_{\Omega} \nabla w \, d\Omega$$

$$\overline{\mathbf{F}} = \overline{\mathbf{F}} + \frac{1}{|\Omega|}\int_{\partial\Omega} w \otimes N \, d\Gamma$$

$$\frac{1}{|\Omega|}\int_{\partial\Omega} w \otimes N \, d\Gamma = 0$$

$$C.L.1: w = 0 \Rightarrow u = (\overline{\mathbf{F}} - \mathbf{I}) X$$

$$C.L.2: w^{+} = w^{-} \Rightarrow u^{+} - u^{-} = (\overline{\mathbf{F}} - \mathbf{I})\left(X^{+} - X^{-}\right)$$

Linear deformation

Periodic condition

Macro problem formulation

The weak form associated with the macroscopic problem is written as:

$$\begin{cases} \int_{\overline{\Omega}} {}^t\overline{\mathbf{P}} : d\,\overline{\mathbf{F}}\, d\Omega = \Lambda \int_{\partial\overline{\Omega}_t} \overline{F}^* . \, d\overline{u}\, d\Gamma \\ \overline{u} = \overline{u}^* \quad sur\ \partial\overline{\Omega}_u \end{cases}$$

The ANM: Asymptotic expansion:

$$\begin{Bmatrix} \overline{u}(a) \\ \overline{\mathbf{P}}(a) \\ \Lambda(a) \end{Bmatrix} = \begin{Bmatrix} \overline{u}_0 \\ \overline{\mathbf{P}}_0 \\ \Lambda_0 \end{Bmatrix} + \sum_{p=1}^{N} a^p \begin{Bmatrix} \overline{u}_p \\ \overline{\mathbf{P}}_p \\ \Lambda_p \end{Bmatrix}$$

Collecting terms with the same powers of ”*a*” induces a sequence of linear problems defined as follows:

$$\int_{\overline{\Omega}} {}^t\overline{\mathbf{P}}_p : d\,\overline{\mathbf{F}}\, d\Omega = \Lambda_p \int_{\partial\overline{\Omega}_t} \overline{F}^* . d\,\overline{u}\, d\Gamma \qquad \forall p = 1 \ldots N$$

Micro problem formulation

The microscopic problem is written as following:

$$\begin{cases} \int_{\Omega} {}^{t}\mathbf{P} : d\,\mathbf{F}\, d\Omega = 0 \\ \mathbf{S} = \mathbf{C}^{(r)} : \boldsymbol{\gamma} \\ \mathbf{P} = \mathbf{F}\,\mathbf{S} \\ \boldsymbol{\gamma} = \tfrac{1}{2}\left({}^{\mathbf{t}}\mathbf{F}\,\mathbf{F} + \mathbf{I}\right) \\ \mathbf{F} = \nabla u + \mathbf{I} \end{cases} \qquad dans \;\; \Omega$$

$$+ C.L. : u^{+} - u^{-} = (\overline{\mathbf{F}} - \mathbf{I})\left(X^{+} - X^{-}\right) \quad sur\ \partial\Omega$$

Micro problem formulation

Perturbation :

Expanding the microscopic problem gives:

At order 1:

$$\begin{cases} \int_\Omega {}^t\mathbf{P}_1 : d\,\mathbf{F}\, d\Omega = 0 \\ \mathbf{S}_1 = \mathbf{C}^{(r)} : \boldsymbol{\gamma}_1 \\ \mathbf{P}_1 = \mathbf{F}_0\mathbf{S}_1 + \mathbf{F}_1\mathbf{S}_0 \\ \boldsymbol{\gamma}_1 = \tfrac{1}{2}\left({}^{\mathbf{t}}\mathbf{F}_0\mathbf{F}_1 + {}^{\mathbf{t}}\mathbf{F}_1\mathbf{F}_0\right) \\ \mathbf{F}_1 = \nabla u_1 \end{cases} \qquad dans\ \ \Omega$$

$$+C.L.: u^+{}_1 - u^-{}_1 = \overline{\mathbf{F}}_1\left(X^+ - X^-\right) \quad sur\ \partial\Omega$$

At order p:

$$\begin{cases} \int_\Omega {}^t\mathbf{P}_p : d\,\mathbf{F}\, d\Omega = 0 \\ \mathbf{S}_p = \mathbf{C}^{(r)} : \boldsymbol{\gamma}_p \\ \mathbf{P}_p = \mathbf{F}_0\mathbf{S}_p + \mathbf{F}_p\mathbf{S}_0 + \sum_{r=1}^{p}\mathbf{F}_r\mathbf{S}_{p-r} \\ \boldsymbol{\gamma}_p = \tfrac{1}{2}\left({}^{\mathbf{t}}\mathbf{F}_0\mathbf{F}_p + {}^{\mathbf{t}}\mathbf{F}_p\mathbf{F}_0 + \sum_{r=1}^{p}{}^{\mathbf{t}}\mathbf{F}_r\mathbf{F}_{p-r}\right) \\ \mathbf{F}_p = \nabla u_p \end{cases} \qquad dans\ \ \Omega$$

$$+C.L.: u^+{}_p - u^-{}_p = \overline{\mathbf{F}}_p\left(X^+ - X^-\right) \qquad sur\ \partial\Omega$$

Micro problem formulation

Substitution:

Using the substitution technique leads to:

At order 1:

$$\begin{cases} L_t(u_1, d\,u) = 0 & dans \;\; \Omega \\ + C.L.: u_1^{\,+} - u_1^{\,-} = \overline{\mathbf{F}}_1 \left(X^+ - X^- \right) & sur \;\; \partial\Omega \end{cases}$$

At order *p*:

$$\begin{cases} L_t(u_p, d\,u) = F_p^{nl}(d\,u) & dans \;\; \Omega \\ + C.L.: u_p^{\,+} - u_p^{\,-} = \overline{\mathbf{F}}_p \left(X^+ - X^- \right) & sur \;\; \partial\Omega \end{cases}$$

Micro problem formulation

Localization:

The linear microscopic problems are solved partially because $\overline{\mathbf{F}}_p$ is unknown. Hence, at order 1, u_1 can be expressed as a linear combination of the solution modes obtained by imposing independently the homogeneous deformation fields through the boundary conditions (e.g. 2D):

$$u_1 = \overline{F}_1^{11}\,\tilde{u}^{(11)} + \overline{F}_1^{12}\,\tilde{u}^{(12)} + \overline{F}_1^{21}\,\tilde{u}^{(21)} + \overline{F}_1^{22}\,\tilde{u}^{(22)} \qquad dans\ \ \Omega$$

where $\tilde{u}^{(ij)}$ are the solutions of the following problem:

$$\begin{cases} L_t\,(\tilde{u}^{(ij)}, d\,u) = 0 & dans\ \ \Omega \\ + C.L.: \tilde{u}^{(ij)^+} - \tilde{u}^{(ij)^-} = X^{(ij)^+} - X^{(ij)^-} & sur\ \ \partial\Omega \end{cases}$$

Micro problem formulation

Therefore, at order *1*:

$$u_1 = \mathbf{A} : \overline{\mathbf{F}}_1$$

where A is a third-order tensor. The microscopic deformation gradient tensor, F, is written as:

Localization tensor

$$\nabla u_1 = \mathbf{F}_1$$

$$u_{1,X} = \nabla u_1 = \mathbf{F}_1 = \boxed{\mathbf{A}_{,X}} : \overline{\mathbf{F}}_1$$

At order *p*, we can obtain in the same manner:

$$u_p = \mathbf{A} : \overline{\mathbf{F}}_p + u_p^{nl}$$

where u_p^{nl} is obtained:

$$L_t(u_p^{nl}, d\,u) = F_p^{nl}(d\,u) \qquad dans \;\; \Omega$$

Hence, we have:

$$\mathbf{F}_p = \mathbf{A}_{,X} : \overline{\mathbf{F}}_p + u_{p,X}^{nl}$$

Micro-macro problem formulation

Therefore, at order *p*: $$\mathbf{P}_p = \mathbf{L} : \overline{\mathbf{F}}_p + \mathbf{P}_p^{nl}$$

The effective stresses can then be obtained at each order *p* by the coupling relation between micro and macro:

$$\overline{\mathbf{P}}_p = \langle \mathbf{L} \rangle : \overline{\mathbf{F}}_p + \langle \mathbf{P}_p^{nl} \rangle$$

$\langle \mathbf{L} \rangle$ **is the macroscopic constitutive tensor.**

Finally, the expansion of macro problem is rewritten:

$$\int_{\overline{\Omega}} {}^t\overline{\mathbf{P}}_p : d\,\overline{\mathbf{F}}\, d\Omega = \Lambda_p \int_{\partial\overline{\Omega}_t} \overline{F}^* . d\,\overline{u}\, d\Gamma$$

$$\Downarrow$$

$$\int_{\overline{\Omega}} {}^t\overline{\mathbf{F}}_p : \langle \mathbf{L} \rangle : d\,\overline{\mathbf{F}}\, d\Omega = \Lambda_p \int_{\partial\overline{\Omega}_t} \overline{F}^* .d\,\overline{u}\, d\Gamma - \int_{\overline{\Omega}} {}^t\langle \mathbf{P}_p^{nl} \rangle :d\,\overline{\mathbf{F}}\, d\Omega$$

ANM for the nonlinear constitutive laws

In the context of ANM, the expansion of the constitutive law can be generally written as following:

Linear constitutive law:

$$\boxed{\mathbf{S}_p = \mathbf{C} : \boldsymbol{\gamma}_p}$$

Nonlinear constitutive law:

$$\boxed{\mathbf{S}_p = \mathbf{C}_t : \boldsymbol{\gamma}_p + \mathbf{S}_p^{res}}$$

Here, two behaviour of materials are considered:

- ***Hyperelasticity***
- ***Plasticity***

ANM for the nonlinear constitutive laws

Isotropic hyperelastic materials:

$$\Psi = \Psi[I_1(\mathbf{C}), I_2(\mathbf{C}), I_3(\mathbf{C})]$$

where

$$I_1 = \mathbf{I} : \mathbf{C} = Tr(\mathbf{C})$$

$$I_2 = \frac{1}{2}\left[Tr(\mathbf{C})^2 - Tr(\mathbf{C}^2)\right]$$

$$I_3 = J^2 = \det(\mathbf{C}) = \det(\mathbf{F})^2$$

$$\mathbf{C} = \mathbf{F}^T\mathbf{F}.$$

where C is the symmetric right Cauchy-Green strain tensor.

ANM for the nonlinear constitutive laws

The second Piola-Kirchhoff stress tensor can be obtained as following:

$$\mathbf{S} = 2\frac{\partial \Psi(\mathbf{C})}{\partial \mathbf{C}} = 2(\frac{\partial \Psi}{\partial I_1}\frac{\partial I_1}{\partial \mathbf{C}} + \frac{\partial \Psi}{\partial I_2}\frac{\partial I_2}{\partial \mathbf{C}} + \frac{\partial \Psi}{\partial I_3}\frac{\partial I_3}{\partial \mathbf{C}})$$

where:

$$\frac{\partial I_1}{\partial \mathrm{C}} = \frac{\partial\left(Tr(\mathrm{C})\right)}{\partial \mathrm{C}} = \frac{\partial\left(\mathbf{I} : \mathrm{C}\right)}{\partial \mathrm{C}} = \mathbf{I}\ ,$$

$$\frac{\partial I_2}{\partial \mathrm{C}} = \frac{1}{2}\left[2Tr(\mathrm{C})\mathbf{I} - \frac{\partial\left(Tr(\mathrm{C}^2)\right)}{\partial \mathrm{C}}\right] = I_1\mathbf{I} - \mathrm{C}\ ,$$

$$\frac{\partial I_3}{\partial \mathrm{C}} = \frac{\partial J^2}{\partial \mathrm{C}} = I_3\mathrm{C}^{-1} = \frac{1}{2}J\mathrm{C}^{-1}\ .$$

Hence, we have:

$$\mathbf{S} = 2\left[\left(\frac{\partial \Psi}{\partial I_1} + I_1\frac{\partial \Psi}{\partial I_2}\right)\mathbf{I} - \frac{\partial \Psi}{\partial I_2}\mathbf{C} + I_3\frac{\partial \Psi}{\partial I_3}\mathbf{C}^{-1}\right]$$

ANM for the nonlinear constitutive laws

Three models of the isotropic compressible hyperelastic materials are considered in this work (for the incompressible case: $J = 1$):

Mooney-Rivlin model:

$$\mathbf{S} = 2(c_1 + c_2 I_1)\mathbf{I} - 2c_2\mathbf{C} + [2cJ(\boxed{J} - 1) - d]\boxed{\mathbf{C}^{-1}}.$$

Neo-Hookean model:

(extension of Hook's law)

$$\mathbf{S} = (\lambda_0 \boxed{\ln J} - \mu_0)\boxed{\mathbf{C}^{-1}} + \mu_0 \mathbf{I}$$

Gent model:

$$\mathbf{S} = \mu \left[\boxed{J}(dJ - d - 1)\boxed{\mathbf{C}^{-1}} + \frac{J_m}{J_m - I_1 + 3}\mathbf{I}\right] .$$

The difficult terms to treat in the context of the ANM are : $\mathbf{C}^{-1}$, J and Ln J.

ANM for the nonlinear constitutive laws

Treatment of the difficulties:

$$\mathrm{C}^{inv} = \mathrm{C}^{-1} \Longrightarrow \mathrm{C}\,\mathrm{C}^{inv} = \mathrm{I} \Longrightarrow$$

$$\mathrm{C}_p^{inv} = -\mathrm{C}_0^{inv}\mathrm{C}_p\mathrm{C}_0^{inv} - \mathrm{C}_0^{inv}\sum_{r=1}^{p-1}\mathrm{C}_r\mathrm{C}_{p-r}^{inv} = -(\mathrm{C}_0^{inv}\overline{\otimes}\mathrm{C}_0^{inv}) : \mathrm{C}_p + \mathrm{C}_p^{inv(nl)}$$

$$J^2 = \det(\mathrm{C}) = \epsilon_{ijk}C_{i1}C_{j2}C_{k3} \Longrightarrow$$

$$2J_0\,J_p = \epsilon_{ijk}\left[(C_{i1}C_{j2})_0(C_{k3})_p + (C_{i1}C_{j2})_p(C_{k3})_0 + \sum_{r=1}^{p-1}(C_{i1}C_{j2})_r(C_{k3})_{p-r}\right] = \mathbb{J} : \mathrm{C}_p + J_p^{nl}$$

$$\beta = \ln J \Longrightarrow J\,\delta\beta - \delta J = 0 \Longrightarrow$$

$$\beta_P = \frac{1}{J_0}\left(J_p - \sum_{r=1}^{p-1}\frac{p-r}{p}J_r\beta_{p-r}\right) =$$

$$\frac{1}{J_0}\mathbb{J} : \mathrm{C}_p + \frac{1}{J_0}\left(J_p^{nl} - \sum_{r=1}^{p-1}\frac{p-r}{p}J_r\beta_{p-r}\right) = \frac{1}{J_0}\mathbb{J} : \mathrm{C}_p + \beta_P^{nl} \;.$$

ANM for the nonlinear constitutive laws

$$\mathbf{S}_p = \mathbf{C}_t : \boldsymbol{\gamma}_p + \mathbf{S}_p^{res}$$

Mooney-Rivlin model:

$$\mathbb{C}_t = 2\left[2c_2\,(\mathbf{I}\,\mathbf{I} - \Im) - \alpha_0(\mathrm{C}_0^{inv}\,\overline{\overline{\otimes}}\,\mathrm{C}_0^{inv})\right] + 4c\,(2J_0 - 1)\,\mathrm{C}_0^{inv}\,\mathbb{J}$$

$$\mathrm{S}_p^{res} = \alpha_0\;\mathrm{C}_p^{inv(nl)} + \alpha_P^{nl}\;\mathrm{C}_0^{inv} + \sum_{r=1}^{p-1}\alpha_r\;\mathrm{C}_{p-r}^{inv}$$

Neo-Hookean model:

(extension of Hook's law)

$$\mathbb{C}_t = -2\,(\lambda_0\beta_0 - \mu_0)\;(\mathrm{C}_0^{inv}\,\overline{\overline{\otimes}}\,\mathrm{C}_0^{inv}) + 2\frac{\lambda_0}{J_0}\mathrm{C}_0^{inv}\mathbb{J}\;,$$

$$\mathrm{S}_p^{res} = (\lambda_0\beta_0 - \mu_0)\;\mathrm{C}_p^{inv(nl)} + \lambda_0\beta_P^{nl}\;\mathrm{C}_0^{inv} + \lambda_0\sum_{r=1}^{p-1}\beta_r\;\mathrm{C}_{p-r}^{inv}\;.$$

Gent model:

$$\mathbb{C}_t = 2\mu\left\{-\kappa_0\,(\mathrm{C}_0^{inv}\,\overline{\overline{\otimes}}\,\mathrm{C}_0^{inv}) + [2dJ_0 - (d+1)]\,\mathrm{C}_0^{inv}\mathbb{J} + \frac{\chi_0}{J_m - (I_1)_0 + 3}\mathbf{I}\,\mathbf{I}\right\}$$

$$\mathrm{S}_p^{res} = \mu\left\{\kappa_0\;\mathrm{C}_p^{inv(nl)} + \kappa_P^{nl}\;\mathrm{C}_0^{inv} + \sum_{r=1}^{p-1}\kappa_r\;\mathrm{C}_{p-r}^{inv}\right\}$$

ANM for the nonlinear constitutive laws

Plasticity:

ØDeformation theory:

Loi de comportement non linéaire Ramberg-Osgood:

$$E\gamma = (1+\upsilon)\mathbf{S}^d - (1-2\upsilon)P\mathbf{I} + \tfrac{3}{2}a\left[\frac{\mathbf{S}_{eq}}{s_y}\right]^{n-1}\mathbf{S}^d$$

Pour plus d'information, voir Thèse de Zahrouni (1998).

ØIncremental theory:

Assidi et al. (2009)

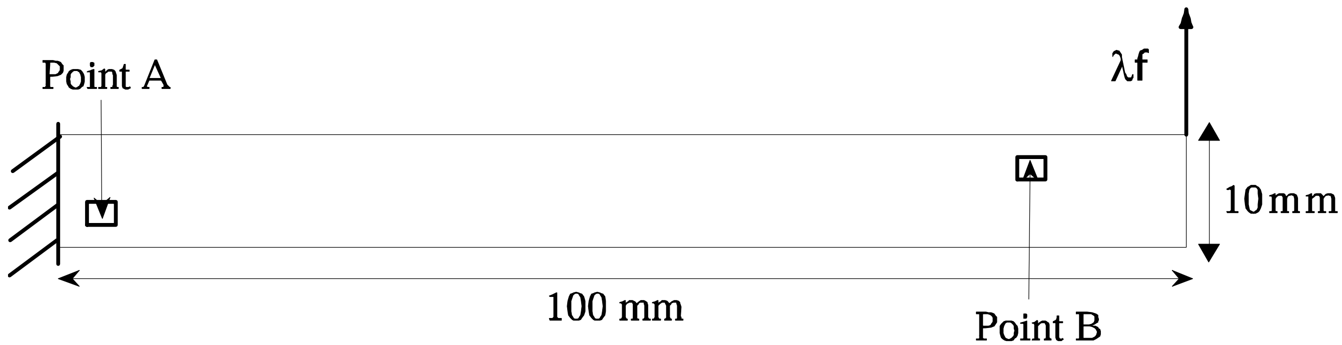
Applications

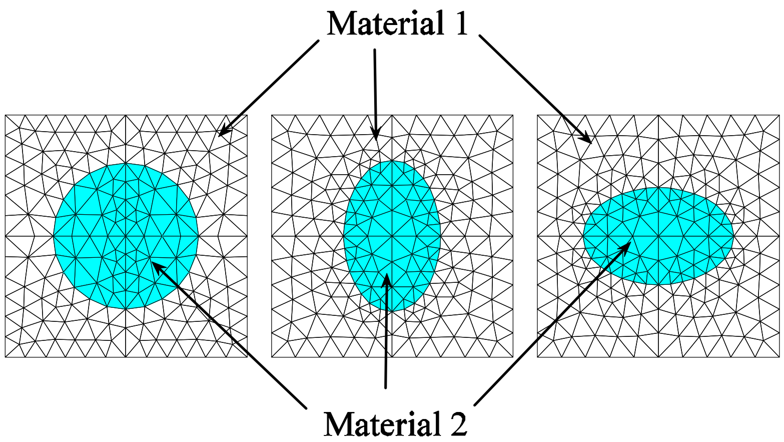
Four examples:

- **Bending of a rectangular beam**
- **Buckling of a shallow arch**
- **Elastic microbuckling of a heterogeneous material**
- **Plastic microbuckling of a fiber reinforced composite**

In all examples:
ANM: $N = 15$ $\delta = 10^{-6}$
Poisson ratio $\nu = 0{,}3$

Bending of a rectangular beam

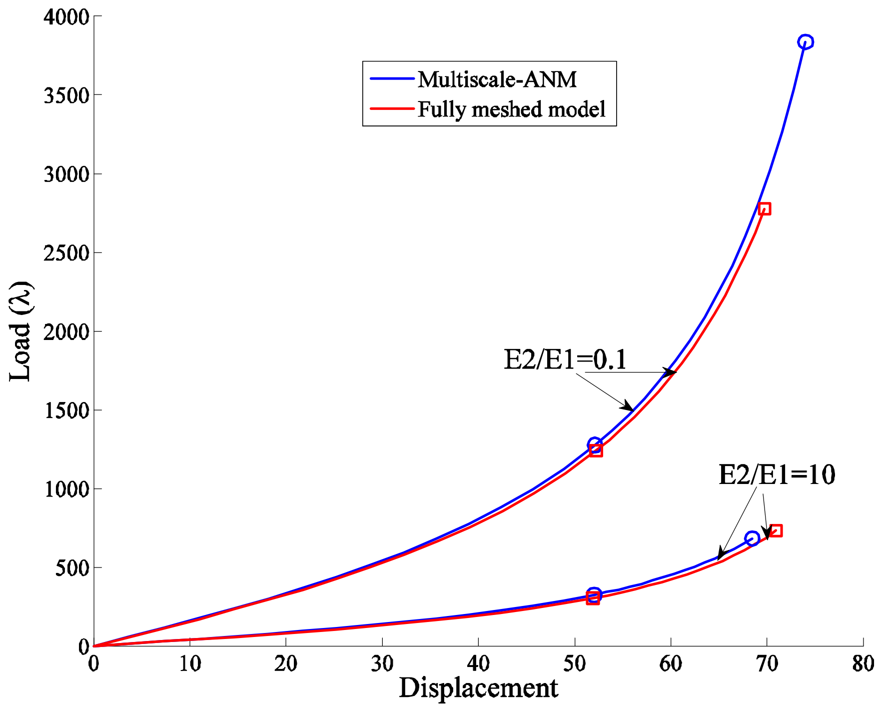


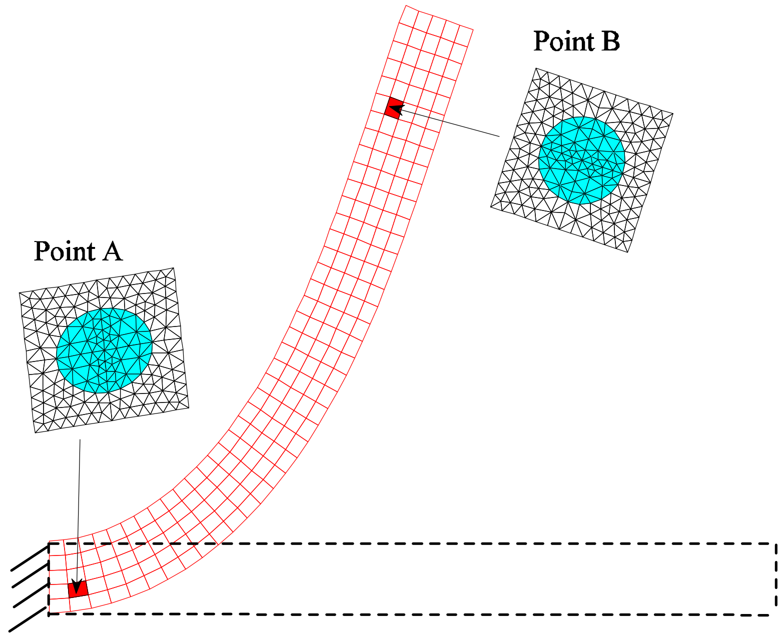


We have tested:

- **Different rigidity ratios;**
- **Different volume fractions;**
- **Different inclusion shapes.**

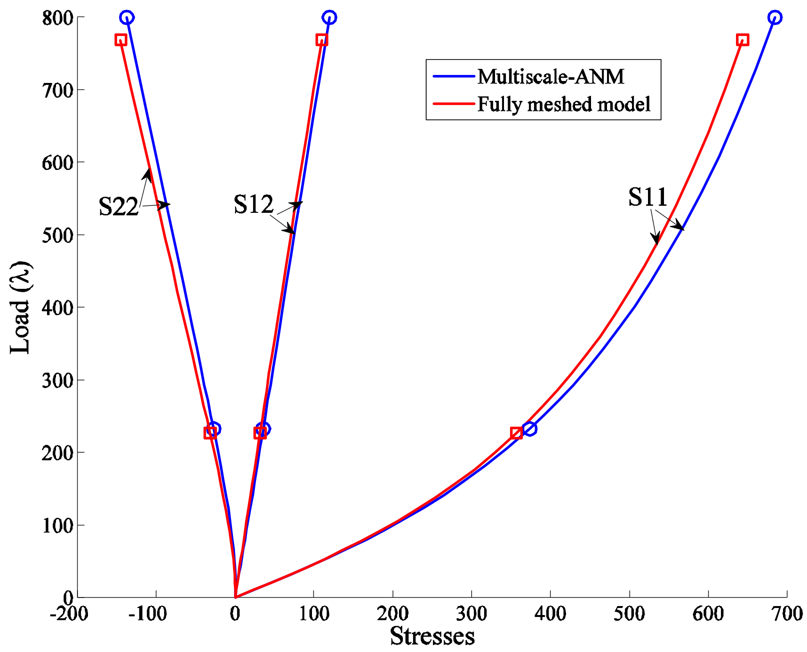
Bending of a rectangular beam



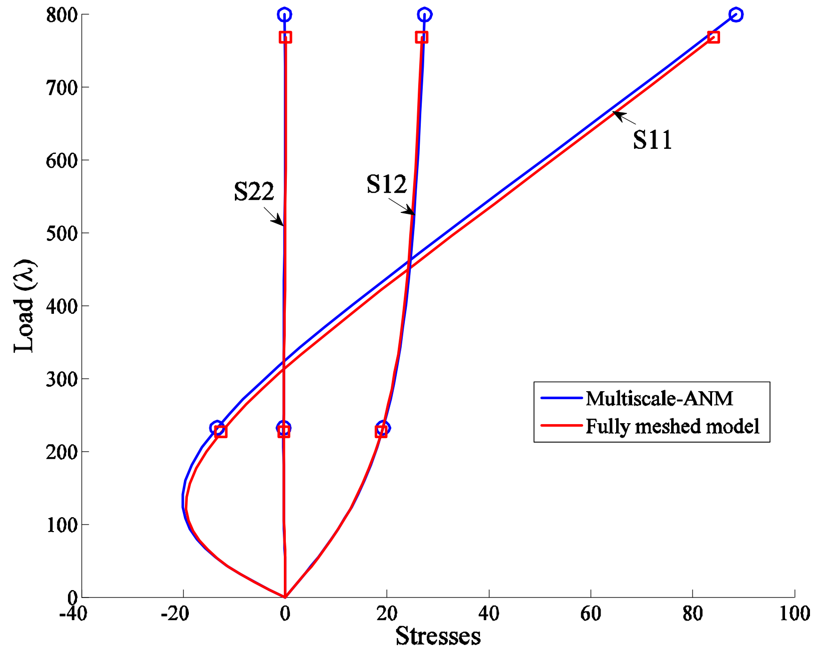


Circular inclusion

Bending of a rectangular beam



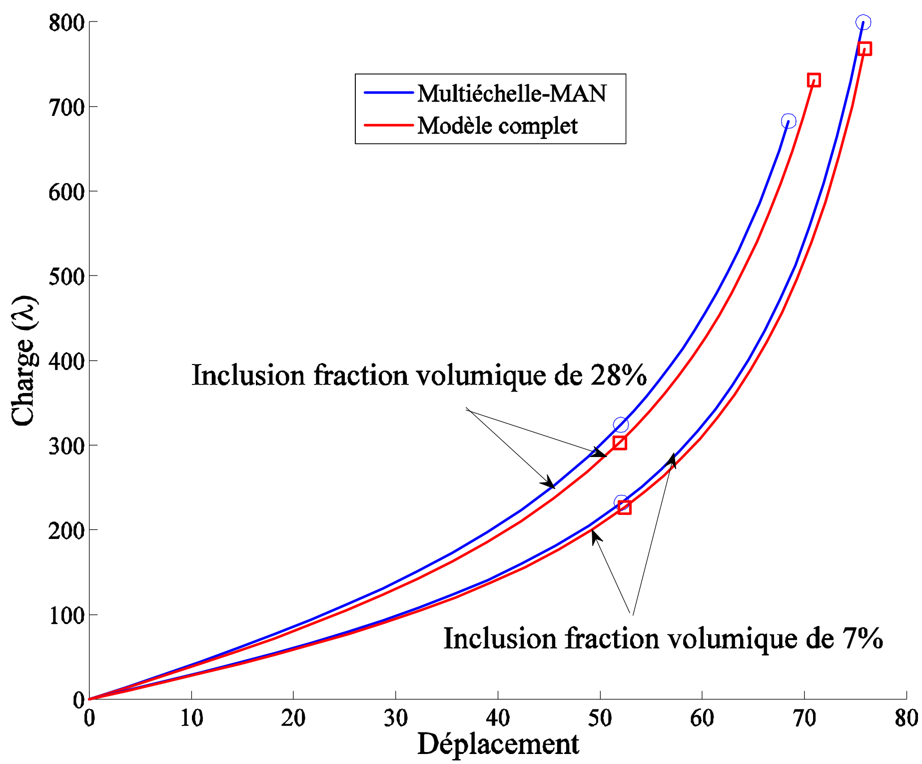
Point A



Point B

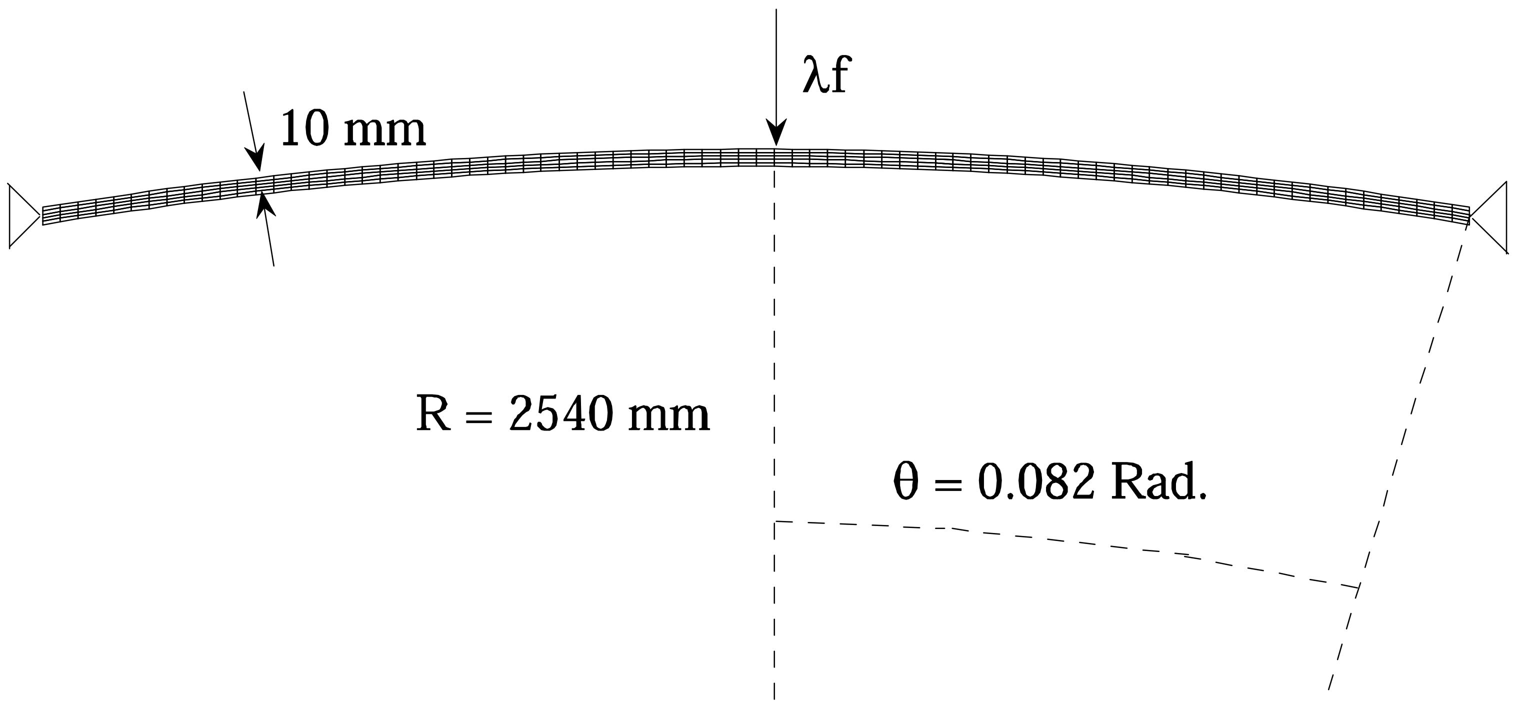
Circular inclusion and rigidity ratio of s E_2/E_1= 10

Bending of a rectangular beam

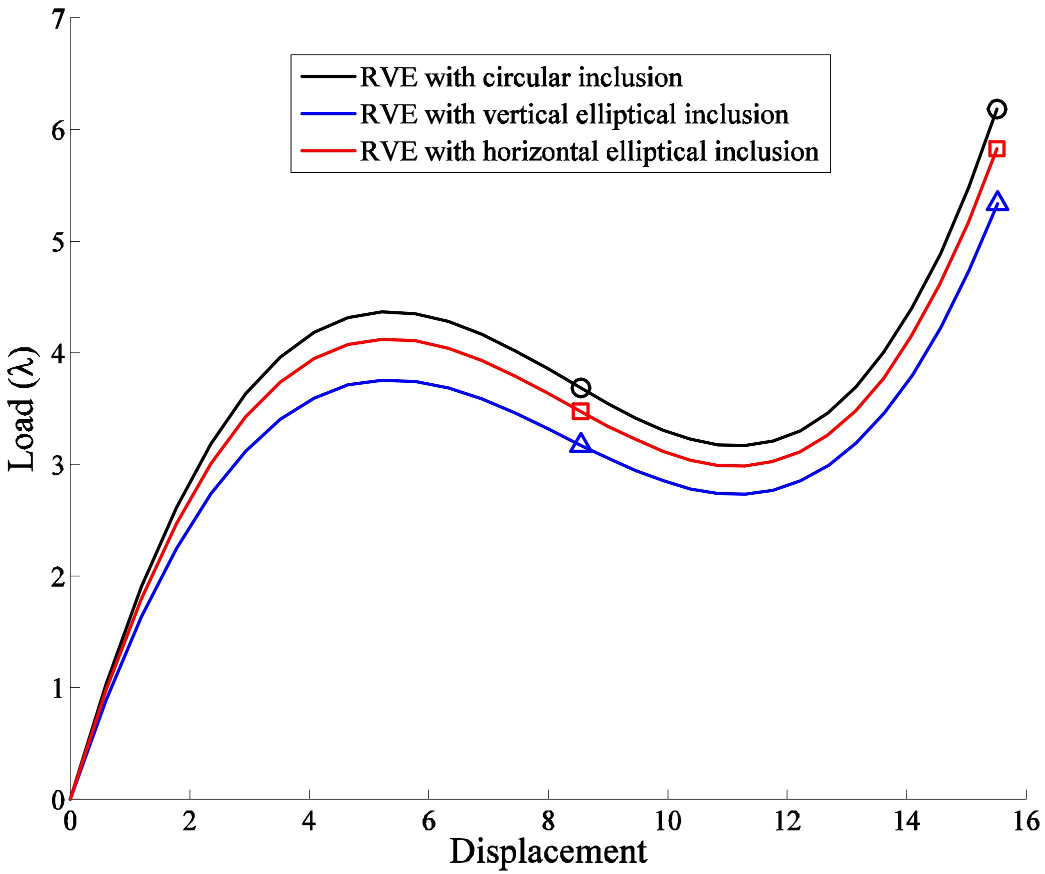


Circular inclusion, E_2/E_1*=10*

Buckling of a shallow arch

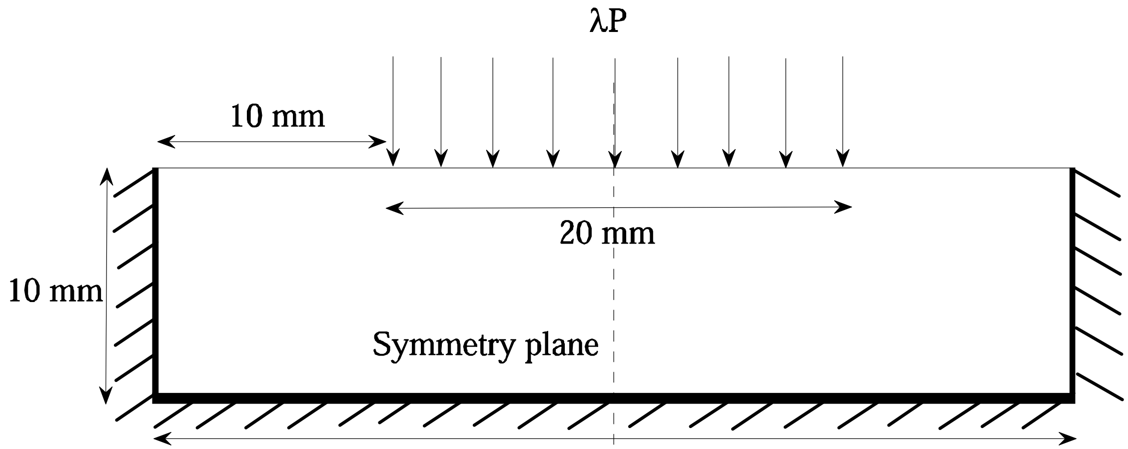


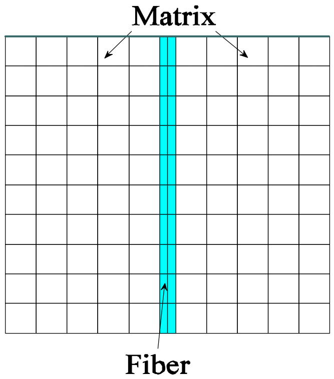
Buckling of a shallow arch



$E_2/E_1=10$

Elastic microbuckling of a heterogeneous material

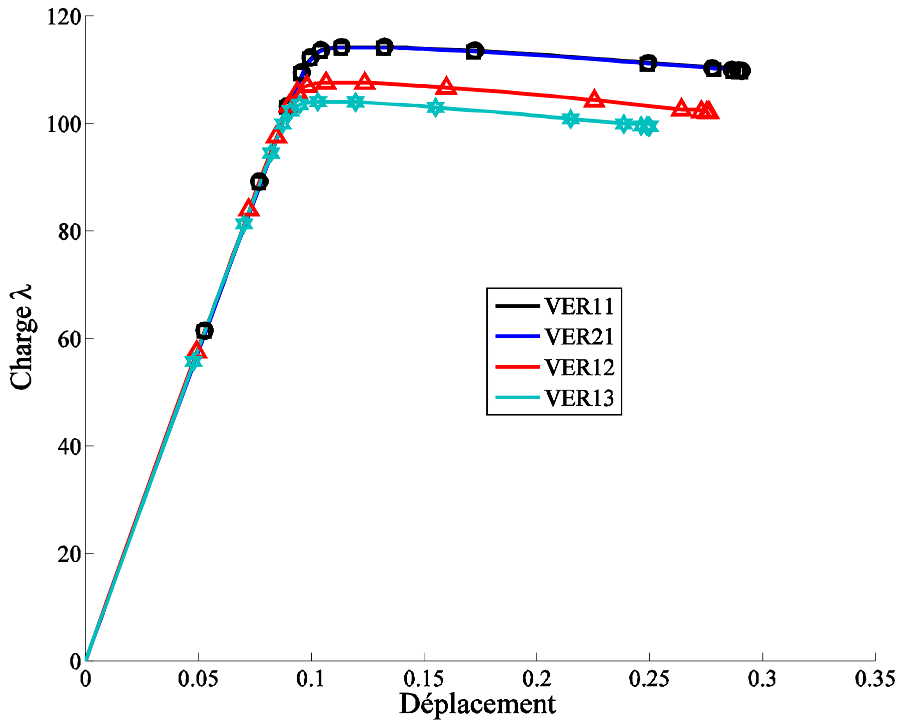


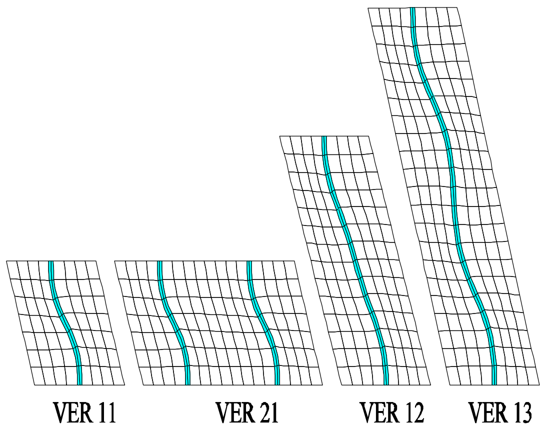


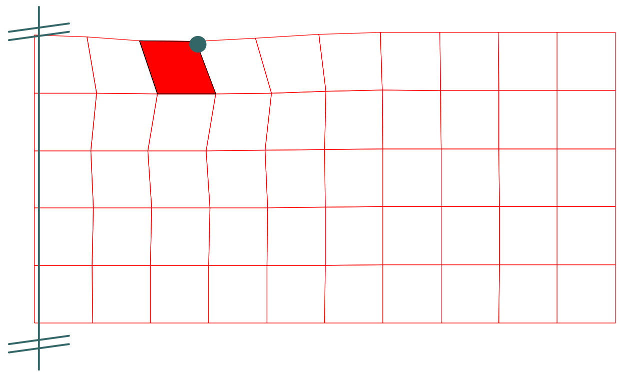
Different RVE can be constructed from the elementary cell.

Elementary Cell (E_{Fiber} /E_{Matrix}=1000)

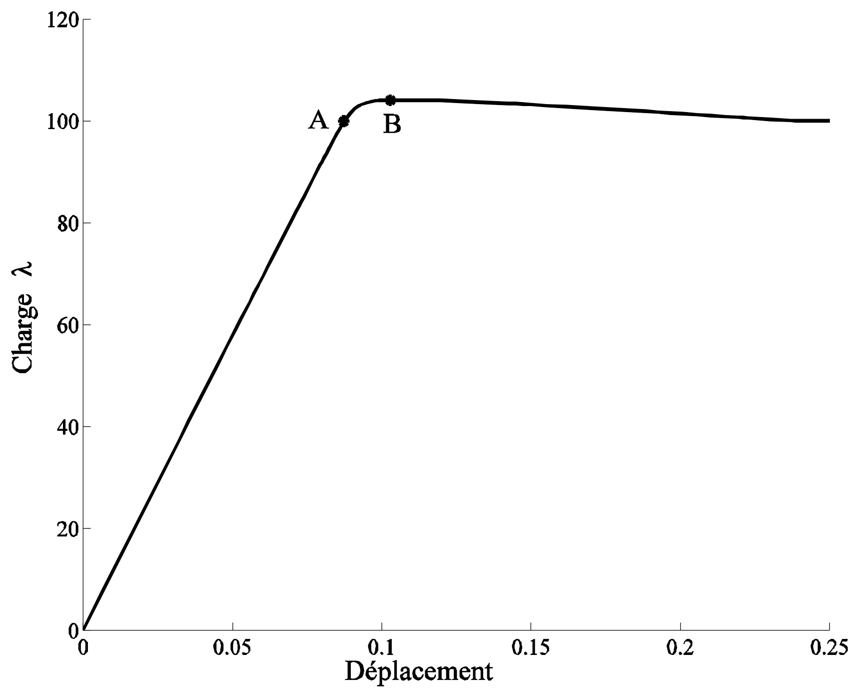
Elastic microbuckling of a heterogeneous material

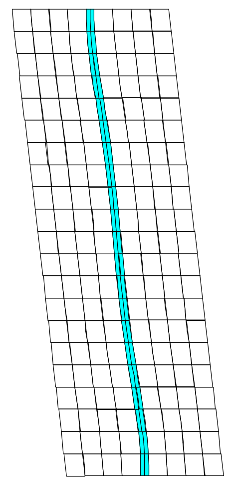




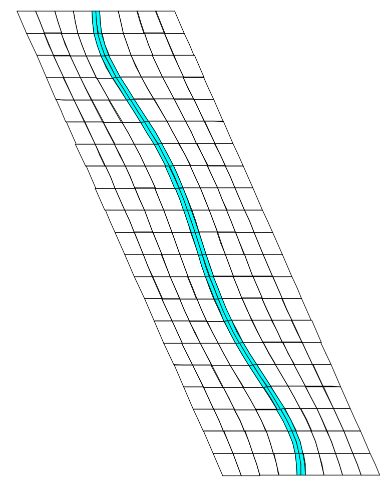


Elastic microbuckling of a heterogeneous material



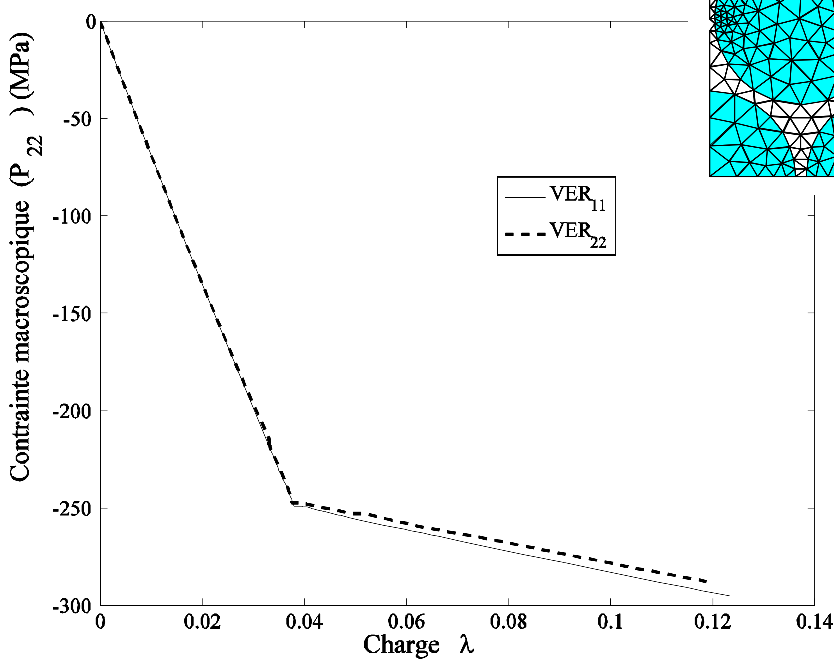


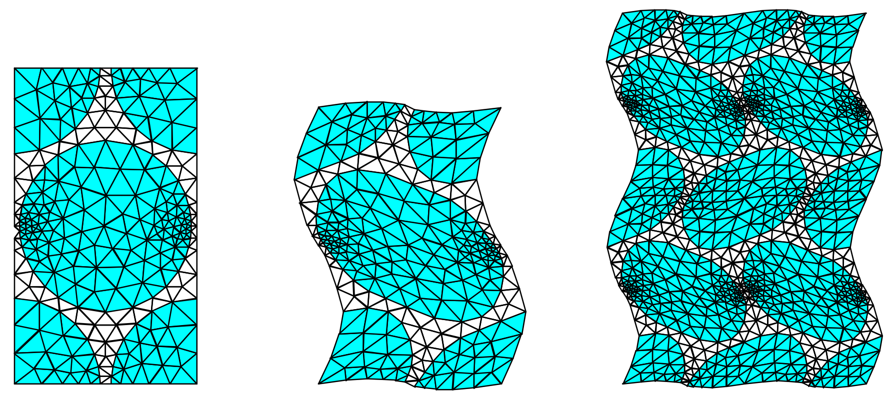
Point A



Point B

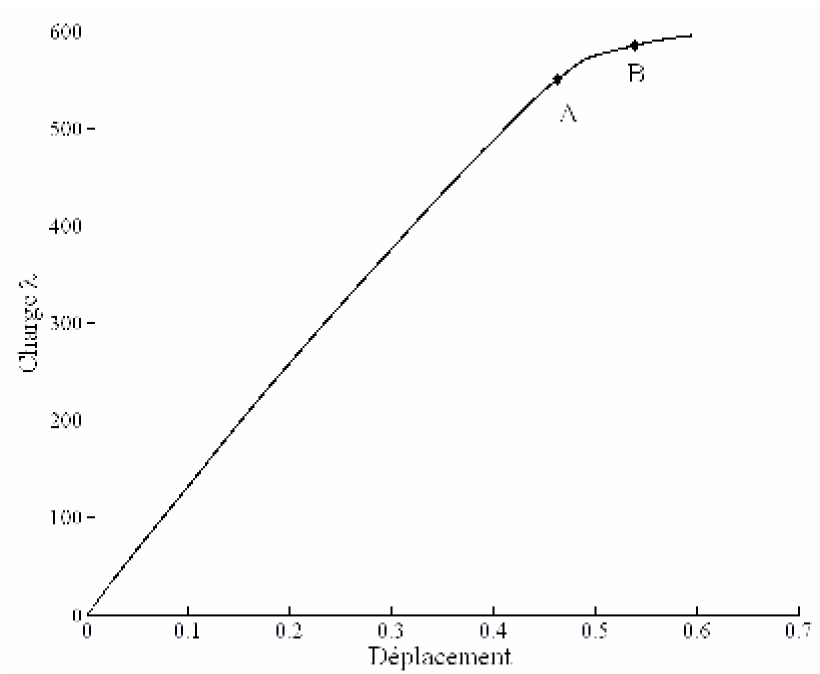
Elastic microbuckling of a heterogeneous material

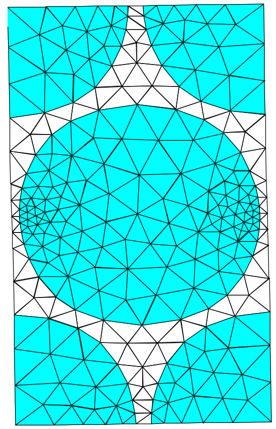




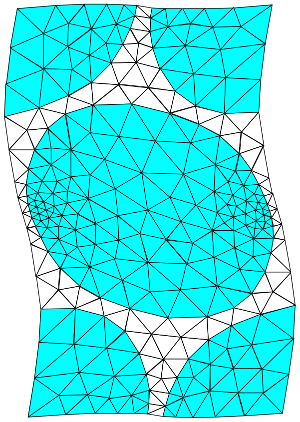
VER_{11} VER_{22}

Elastic microbuckling of a heterogeneous material





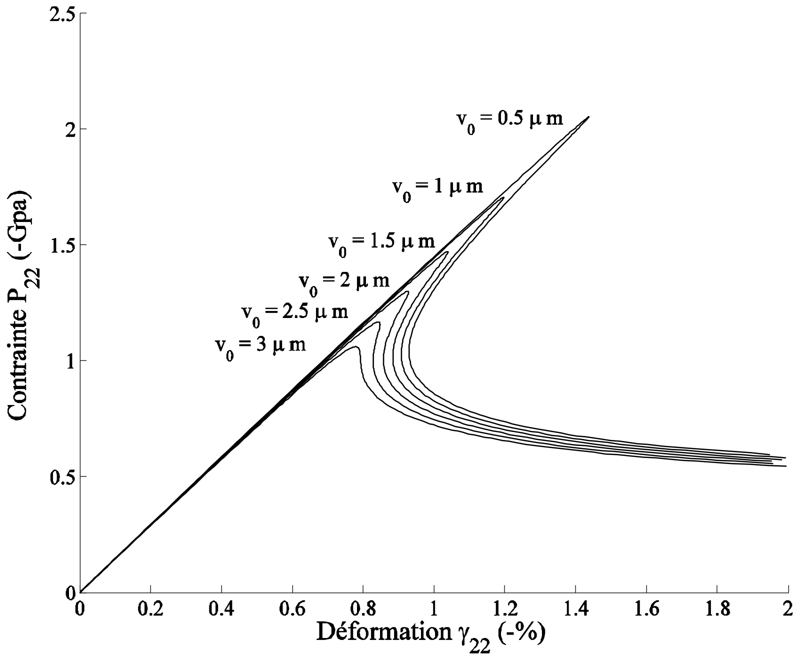
Point A

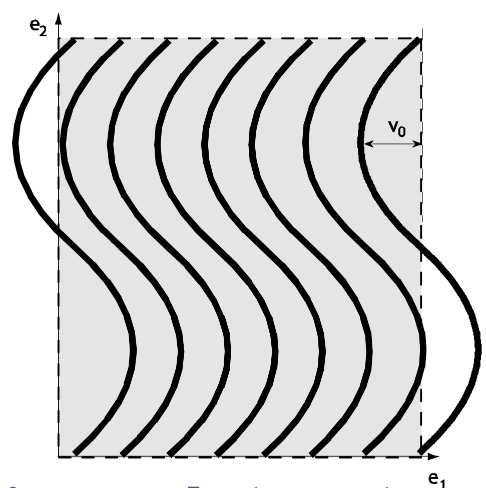


Point B

Plastic microbuckling of a fiber reinforced composite

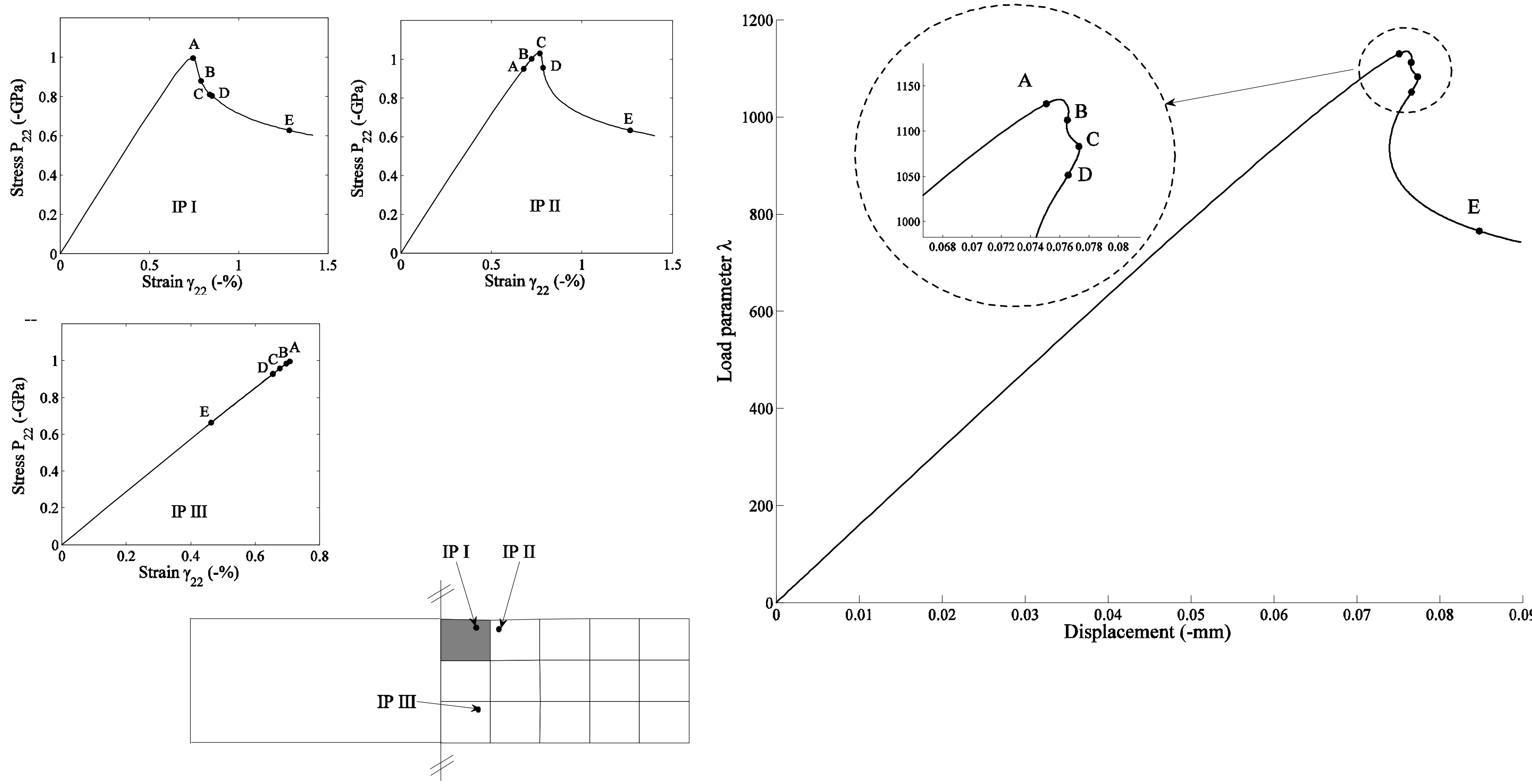
Fibre T300	Matrice 914
$E_f = 240$ Gpa	$E_m = 4.5$ Gpa
$\nu_f = 0.3$	$\nu_m = 0.4$
Fraction volumique de fibre : $f = 0.6$	$n = 6$
Diamètre de fibre : $d_f = 10\mu m$	$\sigma_y = 115$ MPa



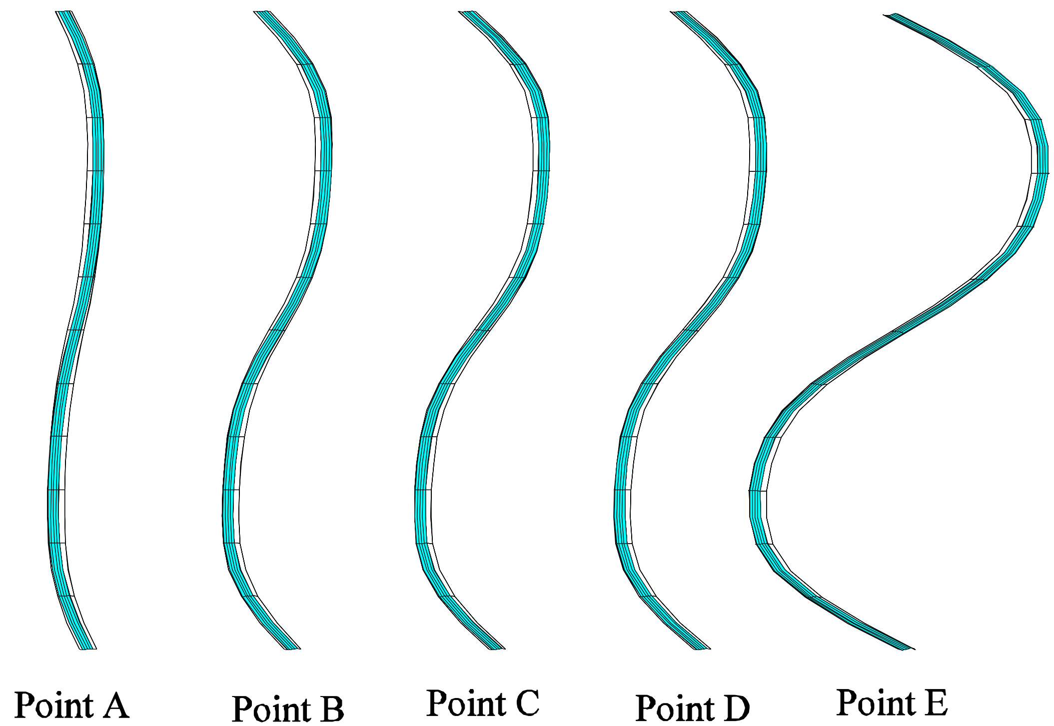


Drapier et al. (1997), (1998) et (2001)

Plastic microbuckling of a fiber reinforced composite



Plastic microbuckling of a fiber reinforced composite



Deformed shapes of the microstructure at IP I

Conclusions

Combining ANM and FE² permits to:

- ***Deal efficiently with the instabilities which may occur on both the micro and macro levels in the heterogeneous materials;***
- ***Apply superposition principle on linear problems obtained by perturbation to construct a localization tensor;***
- ***Large asymptotic step thankful of high order approximation;***
- ***Save computational time compared to the classical Newton-Raphson method.***

Perspectives

- ***Reduction of computational time (Parallel computational, POD, …);***

- ***Second order homogenization:***

$$
{}^{t}\overline{\mathbf{P}} : d\,\overline{\mathbf{F}} + \overline{\mathbf{Q}} \vdots d\,\overline{\mathbf{G}} = \frac{1}{|\Omega|}\int_{\Omega} {}^{t}\mathbf{P} : d\,\mathbf{F}\,d\Omega
$$

$$
x = \overline{\mathbf{F}}X + \tfrac{1}{2}X\,\overline{\mathbf{G}}\,X + w
$$

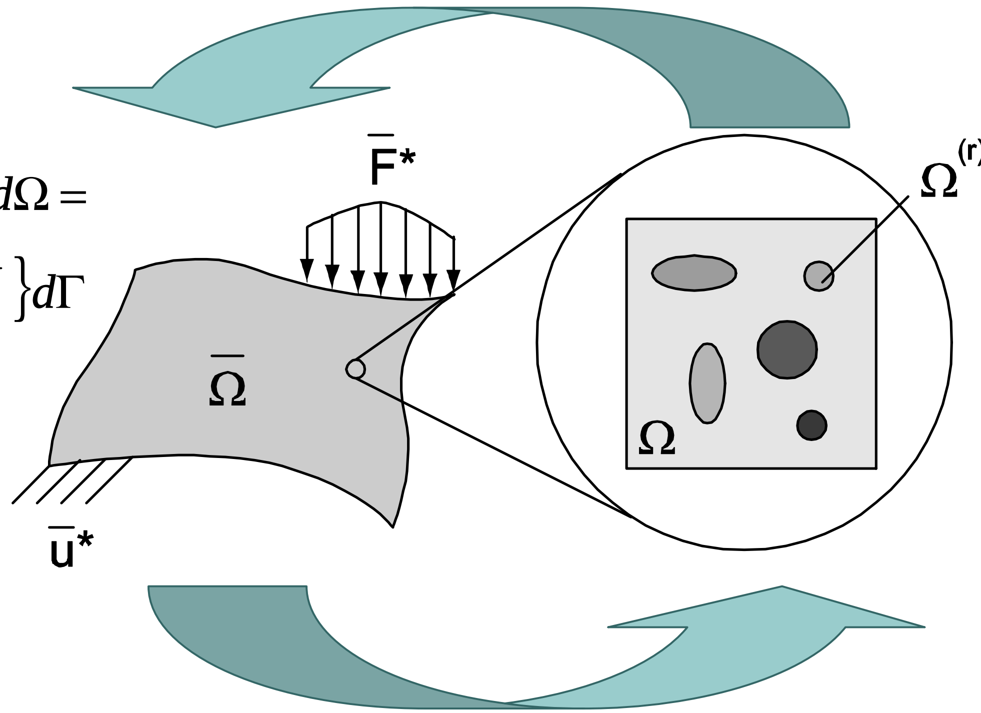
Second order homogenization

$$\overline{\mathbf{P}} = \frac{1}{|\Omega|}\int_{\Omega} \mathbf{P}\, d\Omega \qquad \overline{\mathbf{F}} = \frac{1}{|\Omega|}\int_{\Omega} \mathbf{F}\, d\Omega$$

$$\overline{\mathbf{Q}} = \frac{1}{|\Omega|}\int_{\Omega} \mathbf{Q}\, d\Omega \qquad \overline{\mathbf{G}} = \frac{1}{|\Omega|}\int_{\Omega} \mathbf{G}\, d\Omega$$

$$\int_{\overline{\Omega}} \left\{ {}^t\overline{\mathbf{P}} : d\,\overline{\mathbf{F}} + \overline{\mathbf{Q}} \vdots d\,\overline{\mathbf{G}} \right\} d\Omega =$$

$$\Lambda \int_{\partial\overline{\Omega}_t} \left\{ \overline{F}^* . d\overline{u} + {}^t\mathbf{r} : d\,\overline{\mathbf{F}} \right\} d\Gamma$$



$$\nabla . \mathbf{P} = 0 \quad dans \ \ \Omega$$

$$\mathbf{S} = \mathbf{C}^{(r)} : \boldsymbol{\gamma}$$

Kousenetzova et al.
2002 et 2004

$$\mathrm{x}^+ - \mathrm{x}^- = \bar{\mathrm{F}}\left(\mathrm{X}^+ - \mathrm{X}^-\right) + \frac{1}{2}\bar{\mathrm{G}}\left(\mathrm{X}^+\mathrm{X}^+ - \mathrm{X}^-\mathrm{X}^-\right) \quad sur \ \ \partial\omega$$

$$\int_{\partial\omega^-} \mathrm{x}^- d\Gamma = \bar{\mathrm{F}}. \int_{\partial\omega^-} \mathrm{X}^- d\Gamma + \frac{1}{2}\bar{\mathrm{G}} : \int_{\partial\omega^-} \mathrm{X}^-\mathrm{X}^- d\Gamma$$

Thank you for your attention

S. Nezamabadi, J. Yvonnet, H. Zahrouni and M. Potier-Ferry (2009)
A multilevel computational strategy for microscopic and macroscopic instabilities, *Computer Methods in Applied Mechanics and Engineering*, 198 : 2099-2110.
S. Nezamabadi , H. Zahrouni, J. Yvonnet and M. Potier-Ferry (2009)
A multiscale finite element approach for buckling analysis of elastoplastic long fiber composites, *International Journal for Multiscale Computational Engineering*, to appear.

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