From micro-defects to rupture: coupling asymptotic analysis and strong discontinuity approach.

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ANR Jeunes Chercheurs

Mathématiques et interactions

Context

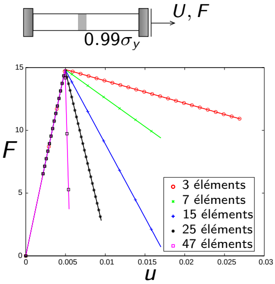
Context

Description of the behavior till rupture of complex structures :

- evaluation of ultimate load
- evaluation of dissipated energy

Treatment of localization process

mesh dependency due to softening behavior



Effect of defects

Initiation and propagation of fracture process

- defects in the loading
- material defects
- geometrical defects

The considered case

Objectives

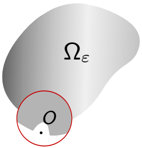
Computation of limit load by taking into account

- the influence of geometrical surface defects
- the influence of localization zones

without fine geometrical description

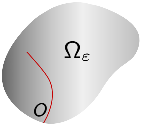
Tools and strategy

Influence of the perturbation
stress concentration



Asymptotic analysis
$$\mathbf{u}_\varepsilon(\mathbf{x}) \simeq \mathbf{u}_0(\mathbf{x}) + \varepsilon\alpha\mathbf{V}(\tfrac{\mathbf{x}}{\varepsilon})$$

Failure description



Strong discontinuity approach
$$\mathbf{u}_{SDA}(\mathbf{x}) = \bar{\mathbf{u}}(\mathbf{x}) + \bar{\bar{\mathbf{u}}}\mathcal{H}_{\Gamma_s}(\mathbf{x})$$

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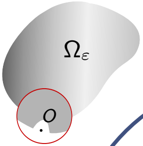
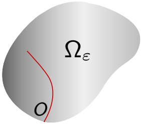


field transfer



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Outline

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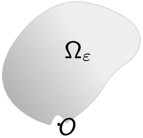
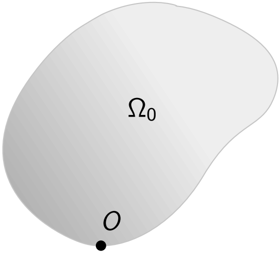
Asymptotic analysis

[M. Dambrine and G. Vial, Control and Cybernetics, 2005]

- **Aim :**
  - evaluate influence of boundary perturbations
  - without fine geometrical description (no fine mesh)



- **The key point : a two scale analysis**
  - the "structural scale" :
    - $\mathbf{u}_0$ solution on Ω_0



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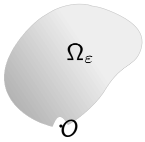

- evaluate influence of boundary perturbations
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The key point : a two scale analysis

- the "structural scale" :
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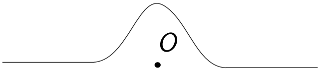
- the scale of the perturbation :

  blow-up around O and introduction of a scaled variable
  - corrector $V(\frac{\mathbf{x}}{\varepsilon})$



Asymptotic analysis

[M. Dambrine and G. Vial, Control and Cybernetics, 2005]

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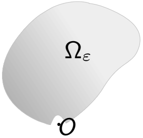

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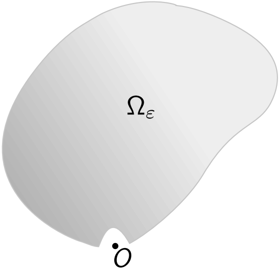


- the "structural scale" :
  - $\mathbf{u}_0$ solution on Ω_0
- the scale of the perturbation :

  blow-up around O and introduction of a scaled variable
  - corrector $V(\frac{\mathbf{x}}{\varepsilon})$
- **Superposition :**
  - solution written as :

$$\mathbf{u}_\varepsilon(\mathbf{x}) = \mathbf{u}_0(\mathbf{x}) + \varepsilon V(\tfrac{\mathbf{x}}{\varepsilon}) + O(\varepsilon)$$

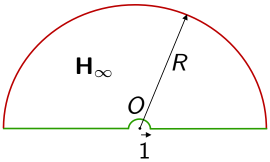
Asymptotic analysis

- For elasticity problems : *[M. Dambrine and G. Vial, Control and Cybernetics, 2005]*

$$\mathbf{u}_\varepsilon(\mathbf{x}) \simeq \mathbf{u}_0(\mathbf{x}) - \varepsilon \left[\boldsymbol{\sigma}_{11}(\mathbf{u}_0(O))\, \mathbf{V}_1(\frac{\mathbf{x}}{\varepsilon}) + \boldsymbol{\sigma}_{12}(\mathbf{u}_0(O))\, \mathbf{V}_2(\frac{\mathbf{x}}{\varepsilon}) \right]$$

- Computation of the «normalised» correctors $\mathbf{V}_\ell(\frac{\mathbf{x}}{\varepsilon})$
  - Solution of Navier equation

$$\begin{cases} \mu\Delta\mathbf{V}_\ell + (\lambda+\mu)\boldsymbol{\nabla}\,\mathrm{div}\,\mathbf{V}_\ell = \mathbf{0} \text{ on } \mathbf{H}_\infty, \\ \sigma(\mathbf{V}_\ell)\cdot\mathbf{n} = \mathbf{G}_\ell \text{ on } \partial\mathbf{H}_\infty, \\ \mathbf{V}_\ell\left(\dfrac{\mathbf{x}}{\varepsilon}\right) \to 0 \quad \text{for } R \to \infty \end{cases}$$



$\mathbf{G}_1 = (\mathbf{n}_1, 0)$ et $\mathbf{G}_2 = (0, \mathbf{n}_1)$, $\mathbf{n} = (\mathbf{n}_1, \mathbf{n}_2)$: outer normal on $\partial\mathbf{H}_\infty$.

  - Computation
    - on the blown-up domain
    - with high order elements (degree 8) with MELINA

Finite element computation

- **Case of several defects :**
  - interaction between perturbations of 2^{nd} order

[Bonnaillie-Noël, Dambrine, Tordeux & Vial, 2009]

if distance $= 2\epsilon^{\alpha}$ with $0 < \alpha < 1$

$$\mathbf{u}_{\varepsilon}(\mathbf{x}) \simeq \mathbf{u}_0(\mathbf{x}) - \sum_{i=1}^{\text{nb defects}} \varepsilon_i \left[\boldsymbol{\alpha}_1^i \mathbf{V}_1^i \left(\frac{\mathbf{x}}{\varepsilon_i} \right) + \boldsymbol{\alpha}_2^i \mathbf{V}_2^i \left(\frac{\mathbf{x}}{\varepsilon_i} \right) \right],$$

with :

$$\alpha_1^i = \sigma_{11}(\mathbf{u}_0)(O_i) \quad , \alpha_2^i = \sigma_{12}(\mathbf{u}_0)(O_i)$$

- **Computation of the solution $\mathbf{u}_{\varepsilon}$**
  - computation of the behavior till rupture : coexistence of the SDA and asymptotic analysis

➥ asymptotic analysis integrated to the computational tool

➥ Local enrichment of the kinematics

Kinematic enrichment

Partition of unity method

[Melenk, Babuska,96], [Moes, Belytschko,99]

- $$\mathbf{u}^h_\varepsilon(\mathbf{x}) = \mathbf{u}^h_0(\mathbf{x}) - \sum_{i=1}^{\text{nb défauts}} \varepsilon_i \sum_{\ell=1}^{2} \sum_{j\in\mathcal{J}_i} N_j(\mathbf{x}) \left[\boldsymbol{\alpha}^i_{j\ell,1} \tilde{\mathrm{V}}^i_{\ell,1}\left(\frac{\mathbf{x}}{\varepsilon_i}\right) + \boldsymbol{\alpha}^i_{j\ell,2} \tilde{\mathrm{V}}^i_{\ell,2}\left(\frac{\mathbf{x}}{\varepsilon_i}\right) \right]$$
  - 4 additionnal dof per node and defect on an enrichment zone around each perturbation
  - if all enriched dofs are free on the enrichment area : bad conditionning
  - master/slave strategy on each enrichment area (suggested by the asymptotic analysis)

$$\left(\alpha^i_{j\ell,1}\right)_1 = \left(\alpha^i_{j\ell,2}\right)_2 \text{ and } \left(\alpha^i_{j\ell,1}\right)_2 = \left(\alpha^i_{j\ell,2}\right)_1$$

Kinematic enrichment

Partition of unity method

[Melenk, Babuska,96], [Moes, Belytschko,99]

- $$\mathbf{u}_\varepsilon^h(\mathbf{x}) = \mathbf{u}_0^h(\mathbf{x}) - \sum_{i=1}^{\text{nb défauts}} \varepsilon_i \sum_{\ell=1}^{2} \sum_{j \in \mathcal{J}_i} N_j(\mathbf{x}) \left[\boldsymbol{\alpha}_{j\ell,1}^i \tilde{\mathrm{V}}_{\ell,1}^i \left(\frac{\mathbf{x}}{\varepsilon_i} \right) + \boldsymbol{\alpha}_{j\ell,2}^i \tilde{\mathrm{V}}_{\ell,2}^i \left(\frac{\mathbf{x}}{\varepsilon_i} \right) \right]$$
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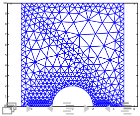
- Resolution of an augmented problem

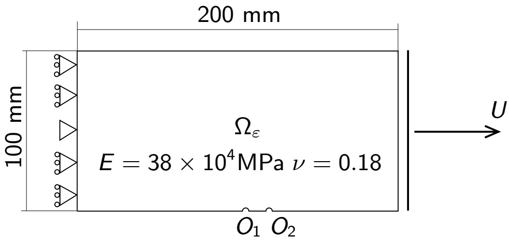
$$\begin{bmatrix} \mathbf{K}_{\mathbf{uu}}^0 & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{\boldsymbol{\alpha}\mathbf{u}}^\varepsilon & \mathbf{K}_{\boldsymbol{\alpha}\boldsymbol{\alpha}}^\varepsilon & \boldsymbol{\Pi}^T \\ \mathbf{0} & \boldsymbol{\Pi} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}_0 \\ \boldsymbol{\alpha} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_0 \\ \mathbf{f}_{\boldsymbol{\alpha}} \\ \mathbf{0} \end{bmatrix}$$

Remark : the computation of $\mathbf{K}_{\alpha\mathbf{u}}^\varepsilon$ and $\mathbf{K}_{\alpha\alpha}^\varepsilon$ requires a specific integration strategy

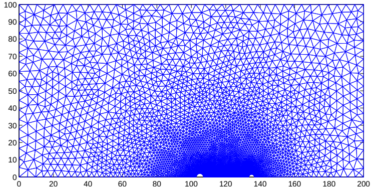


Case of several defects : some results

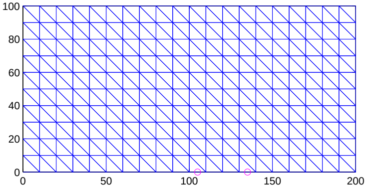
- Simple traction test
  - Comparison with a standard computation on a fine mesh of Ω_ε



(a) Problem geometry $\varepsilon_1 = 2$mm, $\varepsilon_2 = 1$mm

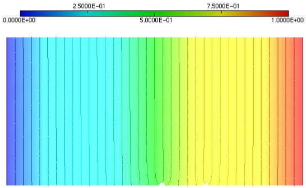


(b) Fine mesh (direct computation)



(c) Coarse mesh (enriched computation)

- Simple traction test
  - Comparison with a standard computation on a fine mesh of Ω_ε



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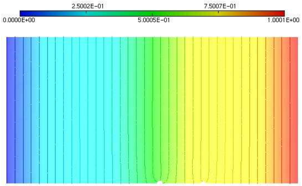

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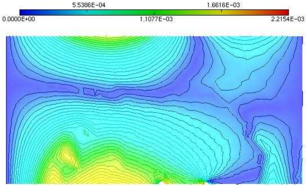

(c) Relative error

displacement u_x and relative error ($< 0.25\%$)

Case of several defects : some results

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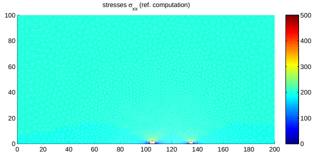

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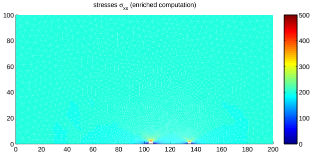

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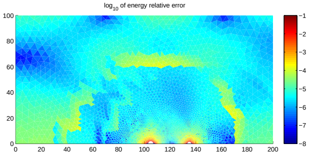
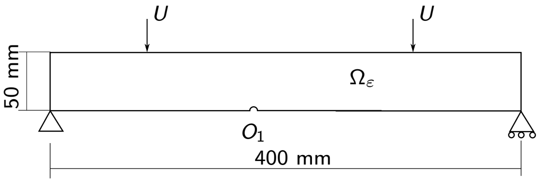


(c) Relative error (strain energy)

stress σ_{xx} and energy relative error (total energy error : $< 0.01\%$)

- Comparison with a standard computation on a fine mesh of Ω_ε



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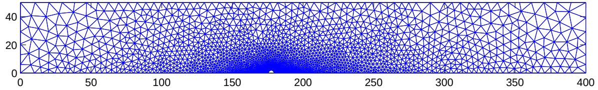

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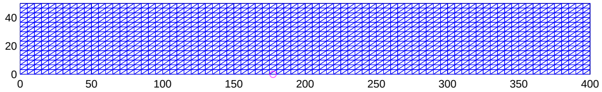
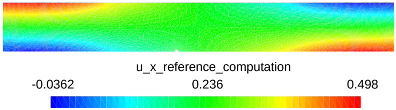


(c) Coarse mesh (enriched computation)

Results : 4-point bending test

- Comparison with a standard computation on a fine mesh of Ω_ε



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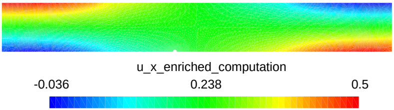
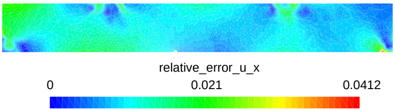


Enriched computation



Relative error

displacement u_x and relative error ($< 4\%$)

Results : 4-point bending test

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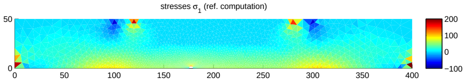

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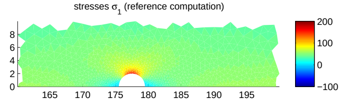

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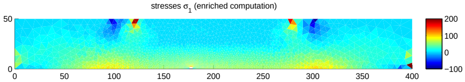

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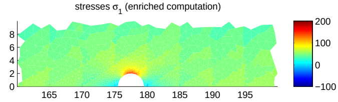

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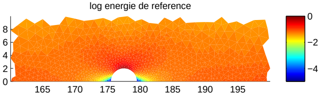
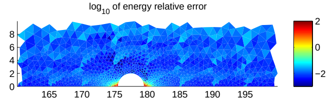


Reference strain energy (log)



Strain energy relative error (log)

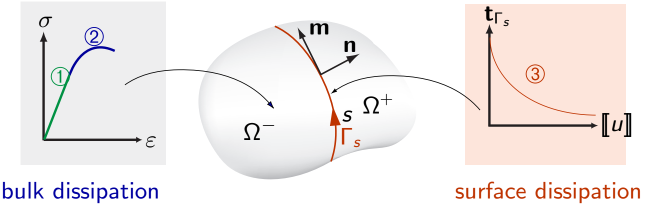
maximal principal stress σ_{I} and relative error ($< 2\%$ around the perturbation)

Outline

Strong discontinuity model

- **Aim** *[Simo et al. 94],[Oliver 96],[Meschke et al. 00],[Jirásek 01],[Wells & Sluys 01]*
  Handle with mesh dependency for localized solutions

- **Key points**
  Description of localization zones (or cohesive cracks) at the structural level by
  - introducing a surface of discontinuity of the displacement field
  - accounting for localised dissipation by a discrete model



No fine description of localized phenomena

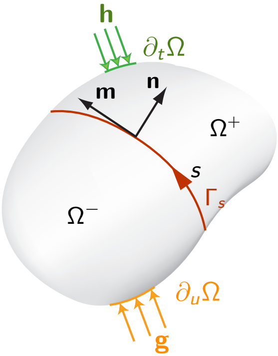
- Equilibrium
  - Equilibrium

    $$\mathbf{div}\,\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \text{ in } \Omega$$

  - Boundary conditions
    - $\boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{h}$ on $\partial_t\Omega$
    - $\mathbf{u} = \mathbf{g}$ on $\partial_u\Omega$
  - Traction continuity

    On all smooth surfaces Γ_s in Ω :

    - $[\boldsymbol{\sigma}]_{|\Gamma_s} \cdot \mathbf{n} = [\mathbf{t}_{\Gamma_s}] = \mathbf{0}$

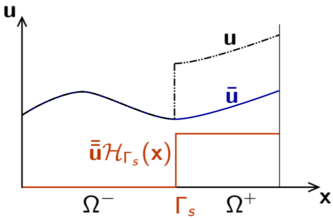


Weak form of the equilibrium :

$$\int_\Omega (\mathbf{div}\,\boldsymbol{\sigma} + \mathbf{b}) \cdot \mathbf{v}\, d\Omega + \int_{\partial_t\Omega} (\mathbf{h} - \boldsymbol{\sigma} \cdot \mathbf{n}) \cdot \mathbf{v}\, d\Gamma + \int_{\Gamma_s} [\boldsymbol{\sigma}]_{\Gamma_s} \cdot \mathbf{n} \cdot \mathbf{v}\, d\Gamma = 0 \quad \forall \mathbf{v} \in CA_0$$

- Displacement field
  - Introduction of a displacement jump $\bar{\bar{\mathbf{u}}}$

    $$\mathbf{u}(\mathbf{x},\,t) = \bar{\mathbf{u}}(\mathbf{x},\,t) + \bar{\bar{\mathbf{u}}}(t)\mathcal{H}_{\Gamma_s}(\mathbf{x})$$

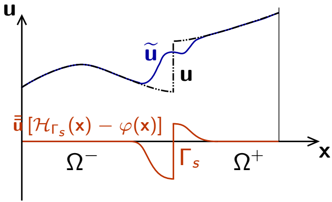


  - Essential boundary conditions :
    a new decomposition

    $$\mathbf{u}(\mathbf{x},\,t) = \tilde{\mathbf{u}}(\mathbf{x},\,t) - \bar{\bar{\mathbf{u}}}(t)\varphi(\mathbf{x}) + \bar{\bar{\mathbf{u}}}(t)\mathcal{H}_{\Gamma_s}(\mathbf{x})$$

  - A multiscale interpretation :

    "macro" scale : $\tilde{\mathbf{u}}(\mathbf{x},\,t)$
    "micro" scale : $\bar{\bar{\mathbf{u}}}(t)\left[\mathcal{H}_{\Gamma_s}(\mathbf{x}) - \varphi(\mathbf{x})\right]$



- Strain field :

$$\varepsilon(\mathbf{x},\,t) = \nabla^s\mathbf{u}(\mathbf{x},\,t) = \nabla^s\bar{\mathbf{u}}(\mathbf{x},\,t) + \left(\bar{\bar{\mathbf{u}}}(t)\otimes\mathbf{n}\right)^s\delta_{\Gamma_s}$$

Theoretical formulation

- Two dissipative mechanisms : two dissipation scales

  Bulk dissipation → Continuum model : $\boldsymbol{\sigma} - \bar{\varepsilon}$ on $\Omega \backslash \Gamma_s$
  Localized dissipation → "Discrete" model : $\mathbf{t}_{\Gamma_s} - \bar{\bar{\mathbf{u}}}$ on Γ_s

- Two similar constructions : Thermodynamics of continuum media and interfaces

  The main ingredients :

Internal and assocated variables	$(\mathrm{V}, \mathbf{A})$
Helmoltz free energy	$\psi(\mathrm{V})$
Yield function	$\phi(\mathbf{A}) \leqslant 0$
Dissipation	$0 \leqslant \mathcal{D}$

▶ State and evolution equations

- Introduction of the discontinuity line :
  - the classical localization condition $\mathbf{A}^{ep} = \mathbf{n} \cdot \mathbf{C}^{ep} \cdot \mathbf{n}$
  - a criterion based on physical considerations (maximal principal stress, maximal shear stress, $\cdots$)

▶ "Instant" t_{loc}
Orientation $\mathbf{n}$

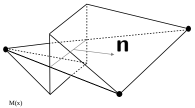
- Displacement interpolation
  - Enrichment of the shape function basis : incompatible modes method

    [Wilson76], [Ibrahimbegovic 91]

    - Continuum formulation : $\mathbf{u}(\mathbf{x},\ t) = \widetilde{\mathbf{u}}(\mathbf{x},\ t) - \bar{\bar{\mathbf{u}}}(t)\varphi(\mathbf{x}) + \bar{\bar{\mathbf{u}}}(t)\mathcal{H}_{\Gamma_s}(\mathbf{x})$

	Real field	Virtual field
displacement	$\mathbf{u}^h(\mathbf{x}, t) = \mathbf{N}\ \mathbf{d} + \bar{\bar{\mathbf{u}}}\ \underbrace{\left(\mathcal{H}_{\Gamma_s} + \bar{\mathbf{M}}\right)}_{\mathbf{M}}$	$\delta\mathbf{u}^h(\mathbf{x}, t) = \mathbf{N}\ \mathbf{w}$
strain	$\varepsilon^h(\mathbf{x}, t) = \mathbf{Bd} + \mathbf{G_r}\bar{\bar{\mathbf{u}}}$	$\delta\varepsilon^h(\mathbf{x}, t) = \mathbf{Bw} + \mathbf{G_v}\bar{\bar{\beta}}$

$$\mathbf{M} = -\sum_{a\in\Omega^+} \mathbf{N}_a(\mathbf{x}) + \mathcal{H}_{\Gamma_s}$$



- Strain interpolation
  - $\mathbf{G_v}$ is chosen to ensure the "patch-test" : $\mathbf{G_v} = \mathbf{G_r} - \dfrac{1}{\Omega}\displaystyle\int_\Omega \mathbf{G_r}\, d\Omega$
  - Strain interpolation functions

$$\mathbf{G_r} = \bar{\mathbf{G}}_\mathbf{r} + \bar{\bar{\mathbf{G}}}_\mathbf{r}\delta_{\Gamma_s} \qquad \text{et} \qquad \mathbf{G_v} = \bar{\mathbf{G}}_\mathbf{v} + \bar{\bar{\mathbf{G}}}_\mathbf{v}\delta_{\Gamma_s}$$

Numerical implementation

- Weak form of equilibrium equation : three field variational formulation

[Hu Washizu 82]

$$\begin{cases} \displaystyle\mathop{\mathrm{A}}_{e=1}^{Nel}\left[\mathbf{f}^{int,e}(t)-\mathbf{f}^{ext,e}(t)\right]=\mathbf{0} & \text{Global equilibrium equation} \\ \displaystyle\mathbf{h}^{e}=\int_{\Omega^e}\bar{\mathbf{G}}_{\mathbf{v}}^{T}\boldsymbol{\sigma}\,d\Omega^e+\int_{\Gamma_s}\bar{\bar{\mathbf{G}}}_{\mathbf{v}}{}^{T}\mathbf{t}_{\Gamma_s}\,d\Gamma_s=\mathbf{0},\forall e\in[1,Nel] & \text{Local equilibrium equation}\end{cases}$$

- Resolution : "operator split" method
  - Local equation

$$Lin(\mathbf{h}^e)=\mathbf{h}^{e\,(i,j)}_{n+1}+\left[\mathbf{H}^{e\,(i,j)}_{n+1}+\mathbf{K}^{(i,j)}_{\alpha\;n+1}\right]\,\Delta\bar{\bar{\mathbf{u}}}^{(i,j)}_{n+1}\Big|_{\mathbf{d}^{(i)}_{n+1}}=\mathbf{0}\quad\blacktriangleright\quad\bar{\bar{\mathbf{u}}}^{(i)}_{n+1}\left(\mathbf{d}^{(i)}_{n+1}\right)$$

  - Global equation

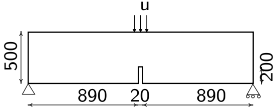
$$\begin{bmatrix}\displaystyle\mathop{\mathrm{A}}_{e=1}^{Nelem}\mathbf{K}^{e\,(i)}_{n+1} & \displaystyle\mathop{\mathrm{A}}_{e=1}^{Nelem}\mathbf{F}_{\mathbf{r}}{}^{e\,(i)}_{n+1} \\ \boxed{\mathbf{F}_{\mathbf{v}}{}^{T^{e\,(i)}}_{n+1}+\mathbf{K}^{(i)}_{d\;n+1} \quad \mathbf{H}^{e\,(i)}_{n+1}+\mathbf{K}^{(i)}_{\alpha\;n+1}}\end{bmatrix}\begin{bmatrix}\Delta\mathbf{d}^{(i)}_{n+1}\\ \boxed{\Delta\bar{\bar{\mathbf{u}}}^{(i)}_{n+1}}\end{bmatrix}=\begin{bmatrix}\mathbf{f}^{e,ext}_{n+1}-\mathbf{f}^{e,int(i)}_{n+1}\\ \boxed{\mathbf{0}}\end{bmatrix}$$

▼ Static condensation at element level

$$\mathop{\mathrm{A}}_{e=1}^{Nelem}\left\{\widehat{\mathbf{K}}^{e\,(i)}_{n+1}\,\Delta\mathbf{d}^{(i)}_{n+1}=\mathbf{f}^{e,ext}_{n+1}-\mathbf{f}^{e,int(i)}_{n+1}\right\}$$

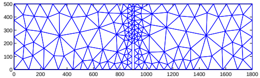
Strong discontinuity model : some results

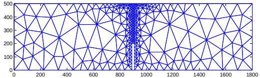
- **3 point bending test : damage model**
  - isotropic damage models (bulk and discontinuity)
  - Introduction of the discontinuity : max. principal stress

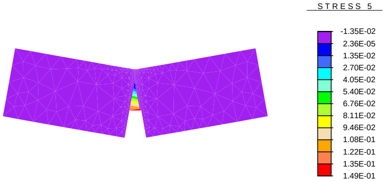


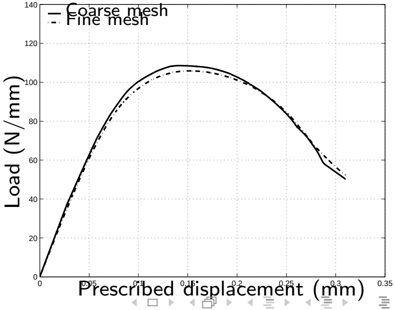
Continuous model	
Young Modulus	38 GPa
Poisson's coefficient	0.1
$\bar{\sigma}_f$	2.2 MPa
K	1000 MPa

Discrete model	
$\bar{\bar{\sigma}}_f$	2.35 MPa
$\bar{\bar{\beta}}_f$	23.5 MPa/mm
$\bar{\bar{\sigma}}_s$	0.235 MPa









Strategy

asymptotic analysis

$\mathbf{u}_\varepsilon^h(\mathbf{x}) \in \mathcal{V}_{AA}$

computation of stress concentration responsible for crack initiation

strong discontinuity approach

$\mathbf{u}^h \in \mathcal{V}_{SDA}$

description of localization and failure

➥ need to project $\mathbf{u}_\varepsilon^h(\mathbf{x})$ on $\mathcal{V}_{SDA}$

Displacement field reconstruction on the coarse "mesh"

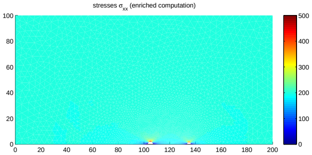
$$\min_{\mathbf{u}\in\mathcal{V}_{SDA}} J(\mathbf{u}) = \mathcal{E}(\mathbf{u} - \mathbf{u}_\varepsilon^h) \quad \text{s.c.} \quad \mathbf{u} = \mathbf{g} \text{ on } \partial_u\Omega$$

with $\mathcal{E}(\mathbf{v}) = \dfrac{1}{2}\displaystyle\int_{\Omega_\varepsilon} \boldsymbol{\sigma}(\mathbf{v}) : \boldsymbol{\varepsilon}(\mathbf{v}) d\Omega.$

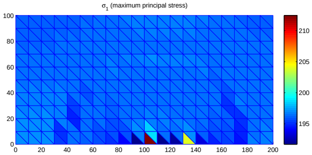
Continuation of the computation : strong discontinuity approach

introduction of the stress field resulting from the projection
as initial value for the "SDA" analysis

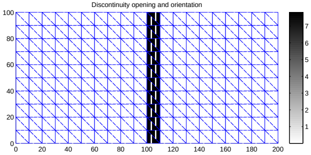
Simple traction test



(a) Enriched computation : initial field



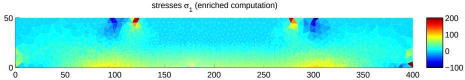
(b) Rebuilt principal stress



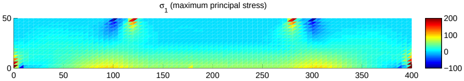
(c) Discontinuities and crack opening

Results

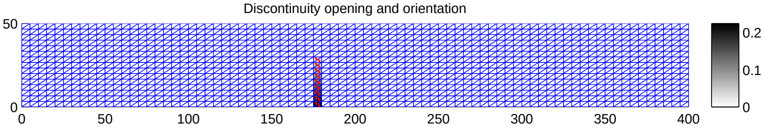
■ 4-point bending test



(a) Enriched computation : initial field



(b) Rebuilt principal stress



(c) Discontinuities and crack opening

Outline

Conclusions and prospects

- **Conclusions**
  - asymptotic analysis of the influence of surface defects
  - strong discontinuity model analysis
  - a first approach to take into account the influence of geometrical defects in localization process

- **Work in progress and prospects**
  - extend the coupling in the case of several defects
  - computation of profile
  - various perturbation geometries
  - randomly distributed (space and length) surface defects
    effect of distribution on localization process